An active anti-rollover device based on Predictive Functional Control: Application to an All-Terrain Vehicle

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Abstract—The active devices dedicated to on-road vehicle stability cannot be applied satisfactorily in an off-road context, since the variability and the non-linear features of grip conditions can no longer be neglected. Specific solutions have then to be investigated. In this paper, the prevention of light All-Terrain Vehicle (ATV) rollover is addressed. First, a backstepping observer is designed in order to estimate online a rollover indicator accounting for sliding phenomena, from a low-cost perception system. Next, the maximum vehicle velocity, compatible with a safe motion over some horizon of prediction, is computed via Predictive Functional Control (PFC), and can then be applied, if needed, to the vehicle actuator to prevent from rollover. The capabilities of the proposed device are demonstrated and discussed thanks to an advanced simulation testbed that has proved to supply results very close to experimental ones.

I. INTRODUCTION

The growing popularity of light All-Terrain Vehicles (ATVs) over the last decade, together with their propensity to rollover, invites to consider the design of on-board safety devices in order to reduce especially lateral rollover fatalities. Indeed, although ATVs may seem harmless at first glance, the number of ATV related deaths and injuries continues to rise. For instance, the Canadian Institute for Health Information reported that hospitalizations related to ATV accidents have increased by 25 per cent over the last decade in Canada (4.104 in 2004 contrary to 3.296 in 1997). In the same time, the Consumer Product Safety Commission (CPSC, [17]) reported, in the U.S.A., 146.600 injuries for the year 2006. Numerous security systems have been developed for road vehicles (active suspensions, active steering [2], steering and braking control [1] and [15]). However, most of these devices do not account for sliding effects. Contrarily, such effects are very significant in ATVs applications and moreover are largely time-varying. Consequently, specific safety devices have to be designed for ATVs.

The first step in the development of such devices is the design of a rollover indicator dedicated to ATVs, including grip condition variations. Previous work [3] has shown that the Lateral Load Transfer (LLT - [7]) is a very relevant criterion. Its advantages, with respect to other stability metrics such as the Static Stability Factor (SSF) [9], the force-angle measurement criterion [11] - [6] or the Zero Moment Point (ZMP - proposed usually to investigate humanoid and mobile robots stability, [14]) are that, on the one hand it does not demand for a huge and expensive perception system (which would be incompatible with ATV applications), and on the other hand it is not dependent on some thresholds particularly difficult to tune in outdoor environment. A backstepping observer, taking into account sliding effects, has then been proposed in [5] in order to estimate on-line the LLT criterion, as well as its expected values on some horizon of prediction, so that imminent rollover situations can actually be detected.

In this paper, this indicator is used as a basis for designing an active anti-rollover device dedicated to ATV. More precisely, the maximum vehicle velocity ensuring that the LLT remains within a safety range over the horizon of prediction is estimated on-line, and can then be applied to the vehicle actuator in order to avoid imminent rollover. The algorithm relies on Predictive Functional Control principle (PFC - [12], [18]) so that ATV dynamic features can be accounted.

The paper is organized as follows: vehicle modeling in presence of sliding is first recalled. Then, the estimation of grip conditions (cornering stiffness) and sliding parameters (sideslip angles), based on a backstepping observer, is shortly described. Next, Predictive Functional Control principle is applied to design vehicle velocity in order to guarantee lateral dynamic stability of ATVs on slippery ground. Finally, advanced simulations of a virtual quad bike are reported and show the relevancy of the proposed approach in situations where lateral rollover is imminent.

II. VEHICLE MODEL AND PREVIOUS WORK

A. Dynamic models

In order to describe the rollover of an All-Terrain Vehicle (ATV), its motion in yaw and roll frames has to be known. As a result, two representations are here introduced: one is a yaw representation (Fig.1(a)) and the other one is a roll representation (Fig.1(b)). The yaw model aims at describing the global vehicle motion on the ground and consists of an extended bicycle model of the ATV. This first part of the model is used to estimate some vehicle motion variables (as the lateral acceleration of the vehicle center of gravity) and sideslip angles (according to a backstepping observer described in Section II-D). These variables are then injected into the second part of the dynamic model, characterized by a roll 2D projection (shown on Fig.1(b)), used to compute roll angle, roll rate and the LLT.

The notations used in this paper, and reported on Fig.1(a) and Fig.1(b), are listed below:

- $R_0(x_0,y_0,z_0)$ is the frame attached to the ground,
- $R_1(x_1,y_1,z_1)$ is the yaw frame attached to the vehicle,
• \( R_2(x_2,y_2,z_2) \) is the roll frame attached to the suspended mass,
• \( \psi \) is the vehicle yaw angle,
• \( \phi_r \) is the roll angle of the suspended mass,
• \( \delta \) is the steering angle,
• \( \beta, \alpha_r, \alpha_f \) are the global, rear and front sideslip angles,
• \( v \) is the linear velocity at the center of the rear axle,
• \( u \) is the linear velocity at the roll center,
• \( a \) and \( b \) are the front and rear vehicle half-wheelbases,
• \( L = a + b \) is the vehicle wheelbase,
• \( c \) is the vehicle track,
• \( h \) is the distance between the roll center \( O' \) and the vehicle center of gravity \( G \),
• \( I_x, I_y, I_z \) are the roll, pitch and yaw moments of inertia,
• \( P = mg \) is the gravity force on the suspended mass \( m \), with \( g \) denoting the gravity acceleration,
• \( F_y \) and \( F_r \) are the front and rear lateral forces,
• \( F_{n1} \) and \( F_{n2} \) are the normal component of the tire/ground contact forces on the vehicle left and right sides,
• \( F_a \) is a restoring-force parametrized by \( k_r \) and \( b_r \), the roll stiffness and damping coefficients:

\[
\overrightarrow{F_a} = \frac{1}{h}(k_r\phi_r + b_r\dot{\phi}_r)\ y_2
\]  

(1)

The roll stiffness \( k_r \) and the distance \( h \) are assumed to be preliminary calibrated, as explained in Section IV-A. The roll damping \( b_r \) is experimentally evaluated (through a driving procedure) and the other parameters (wheelbase, weight, etc) are directly measured.

Based on (2), the dynamic equations of the yaw model (see [16]) can be expressed as:

\[
\begin{align*}
\dot{\psi} & = \frac{1}{h}(aC_f \alpha_f \cos(\delta) + bC_r \alpha_r) \\
\dot{\beta} & = -\frac{1}{um}(C_f \alpha_f \cos(\beta - \delta) + C_r \alpha_r \cos(\beta)) - \dot{\psi} \\
\dot{\alpha_r} & = \arctan\left(\frac{\tan(\beta) - \frac{h\psi}{ucos(\beta)}}{\frac{a\psi}{ucos(\beta)}}\right) \\
\dot{\alpha_f} & = \arctan\left(\frac{\tan(\beta) + \frac{uv}{ucos(\beta)}}{\frac{a\psi}{ucos(\beta)}}\right) - \delta \\
u & = \frac{vcos(\alpha_r)}{cos(\beta)}
\end{align*}
\]  

(3)

C. Lateral Load Transfer computation

1) LLT definition: The general expression of the Lateral Load Transfer (LLT) (see [8], [1]) is:

\[
LLT = \frac{F_{n1} - F_{n2}}{F_{n1} + F_{n2}}
\]  

(4)

Clearly, a rollover situation is detected when a unitary value of \( |LLT| \) is reached, since it corresponds to the lift-off of the wheels on the same side of the vehicle. Here, the vehicle behavior will be considered as hazardous when LLT reaches the critical threshold 0.8.

2) LLT dynamic equations: In order to extract normal force expressions from the roll model (see Fig.1(b)), the following assumptions have been made:

• The entire vehicle mass is suspended, which implies insignificant non-suspended mass (essentially tires).
• The suspended mass is assumed to be symmetrical with respect to the two planes \((z_2, y_2)\) and \((x_2, z_2)\). The inertial matrix is then diagonal:

\[
I_{G/z_2} = \begin{bmatrix}
I_x & 0 & 0 \\
0 & I_y & 0 \\
0 & 0 & I_z
\end{bmatrix}
\]  

(5)

• Sideslip angles \( \alpha_f, \alpha_r \) and \( \beta \) are assumed to be small (corroborated by experiments),
• As a consequence, the vehicle velocity \( u \) at roll center can be considered to be equal to the rear axle one (i.e. \( u \approx v \)), see (3).

Using these assumptions, the LLT indicator can be evaluated from the Fundamental Principle of the Dynamic (FPD) applied to the overall system, subjected to four external forces \((P, F_a, F_{n1} \text{ and } F_{n2})\). More precisely, variations of \( \phi_r, F_{n1} \text{ and } F_{n2} \) can be derived as:

\[
\begin{align*}
\phi_r & = \frac{1}{h\cos(\phi_r)}[h\phi_r^2\sin(\phi_r) + h\dot{v}^2\sin(\phi_r) + u\psi\cos(\beta) + \\
& \quad \dot{u}\sin(\beta) + u\dot{\beta}\cos(\beta) - \left(\frac{k_r\phi_r + b_r\dot{\phi}_r}{mh}\right)\cos(\phi_r)] \\
F_{n1} + F_{n2} & = m \left[-h\phi_r\sin(\phi_r) - h\phi_r^2\cos(\phi_r) + g \\
& \quad - \left(\frac{k_r\phi_r + b_r\dot{\phi}_r}{mh}\right)\sin(\phi_r)\right] \right)
\]  

(6)

\[
F_{n1} - F_{n2} = \frac{2}{c}[I_x\phi_r + (I_z - I_x)\left[\psi^2\cos(\phi_r)\sin(\phi_r)\right] \\
& \quad - h\sin(\phi_r)(F_{n1} + F_{n2})]
\]  

(7)

In order to infer the roll angle and the LLT from (6)-(8), the global sideslip angle and the yaw rate are both required.
Since the former one cannot be measured, an observer has been designed and is presented below.

D. Backstepping observer

In order to account for both tire/ground contact non-linearities and grip condition variability, a backstepping observer has been proposed in previous work [4] and [5]. It can be summarized by the scheme depicted on Fig.2.

The only three available measurements are the yaw rate $\dot{\psi}$ (available from a gyrometer), the rear axle linear velocity $v$ (from a Doppler radar) and the steering angle $\delta$ (from a steering angle sensor). These three variables do not permit to estimate $C_f$ and $C_r$ separately. As a result, they are here considered to be equal to a virtual tire cornering stiffness $C_e$.

The backstepping observer is divided into two steps. The first one consists in computing a virtual measurement of the global sideslip angle (noted $\beta$ on Fig.2). More precisely, $\beta$ is derived by imposing the convergence of the estimated yaw rate $\hat{\psi}$ to the measured one $\psi$.

This virtual global sideslip angle $\beta$ is then treated as a reference to be reached by the observed sideslip angle $\hat{\beta}$. This is ensured by designing an adaptation law on $C_e$. As mentioned in previous work [4] and [5], this observer is stable and ensures asymptotic convergence except when the vehicle is at stop ($v = 0$), which is not considered here, or in the vicinity of neutral steer ($\delta = 0$). Close to this latter situation, the virtual cornering stiffness is not adapted but kept equal to its previous value.

III. PREDICTIVE FUNCTIONAL CONTROL OF ATV VELOCITY

A. Strategy of LLT limitation

In order to avoid the rollover risk, the limitation of the LLT (i.e. $LLT \leq 0.8$) through the control of the ATV speed is here investigated. The idea is to compute at each time the velocity leading to this LLT threshold one moment in the future. This value can then be considered as the maximum admissible velocity (denoted $v_{max}$ in the sequel) to avoid lateral rollover situation.

The global scheme is depicted on Fig.3. The computation of the maximum velocity, detailed in Section III-C, is represented by the block “Predictive control”. Relying on this variable, the speed limitation process consists then on the following steps:

- The “Min” block supplies the rear axle linear velocity control input $v_{input}$ to be applied to the vehicle. This variable is deduced from the comparison between the velocity specified by the pilot $v_{pilot}$ and the maximum velocity $v_{max}$: $v_{input} = \min(v_{pilot}, v_{max})$
- The three measurements shown on Fig.3 are then used to estimate on-line the sliding parameters and the global cornering stiffness thanks to the backstepping observer described in Section II-D.
- Finally, the roll angle $\phi_v$, the sliding parameters and the steering angle are processed in the “Predictive Control” block in order to supply the maximum velocity $v_{max}$.

In order to anticipate (and then avoid) hazardous situations, the computation of $v_{max}$ is based on the Predictive Functional Control (PFC) formalism, detailed in [12] and [18]. The vehicle velocity is then viewed as a control variable and $v_{max}$ is designed in order to ensure the convergence of the LLT to the value 0.8.

B. Roll angle model

As it can be seen in equations (7) and (8), the LLT does not rely explicitly on vehicle velocity, but on roll angle: vehicle velocity should then be designed to control $\phi_v$.

The roll angle equation (6) is non-linear when PFC formalism requires linear equations, see [13]. Therefore, as a first step it is necessary to approximate equation (6) to a linear model. In the sequel, $\phi_{vNL}$ and $\phi_{vL}$ denote the roll angle supplied respectively by non-linear model (6) and by the linear model to be derived.

In order to achieve the linearization, the following assumptions are considered:

- Sideslip angles are quite small and consequently, based on (3), the vehicle yaw rate can be approximated by:

$$\dot{\psi} = u \left( \frac{\dot{\delta} + \alpha_f - \alpha_r}{L} \right)$$  \hspace{1cm} (9)

- Since $\beta$ and $u$ are slow-varying with respect to $\dot{\psi}$, terms $u\dot{\beta}\cos(\beta)$, $u\dot{\beta}\sin(\beta)$ are widely negligible with respect to $u\dot{\psi}\cos(\beta)$ (corroborated by advanced simulations and experiments).

Linearization of (6) around $(\phi_v, \dot{\phi}_v)=(0,0)$ then leads to:

$$\dot{\phi}_v = \frac{1}{mh} \left[ u^2 \cos(\beta) \left( \frac{\delta + \alpha_f - \alpha_r}{L} \right) - \frac{kr\phi_{vL} + br\phi_{vL}}{mh} \right]$$  \hspace{1cm} (10)

Since $u \equiv v$ (as previously mentioned), the linear state-space model to be used in PFC algorithm is then:

$$\begin{cases}
\dot{X} = AX + Bw \\
Y = CX
\end{cases}$$  \hspace{1cm} (11)
with the state-space vector $X = (\phi v_L, ˙\phi v_L)^T$, the control variable $w = v_L^2$ and matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k_v}{mbz} & -\frac{b_v}{mbz^2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \cos(\beta) (\delta + \alpha - \alpha L) \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$ 

Based on Kalman criterion, the controllability of model (11) can be established provided that $ψ \neq 0$. In other words, the linear roll angle $\phi v_L$ cannot be controlled when the vehicle is moving in straight line, which is quite natural. Then, close to neutral steering ($|\delta|$ below some steering limit), the PFC control algorithm is not activated and $v_{input} = v_{pilot}$.

C. Predictive maximum velocity computation

PFC algorithm is now applied to linear system (11) in order to derive the maximum velocity $v_{max}$. The principle of the predictive approach is summarized on Fig.4. Roughly, it consists in finding the control sequence which permits to reach “at best” the future set point after a specified horizon of prediction $H$.

![Fig. 4. Prediction principle.](image)

More precisely, the algorithm consists in the following steps:

- The first step consists in computing the roll angle value, hereafter denoted $\phi_{target}$, leading to a $LLT$ steady state value equal to the critical threshold 0.8. Relying on the following assumptions: $\delta = \phi = 0$ and $ξ_1 = (L - L_f)[ψ^2 \cos(\phi_0) \sin(\phi_0)]$ is widely negligible with respect to $ξ_2 = h \sin(\phi_1)(F_{n1} + F_{n2})$ (in view of quad bike properties - see Table I - and actual conditions, the magnitude of $ξ_1$ stays beyond $100 \sin(\phi_0)$ while the magnitude of $ξ_2$ is at least equal to $3000 \sin(\phi_0)$) it can be derived from equations (7) and (8) that:

$$|LLT| = \frac{|F_{n1} - F_{n2}|}{F_{n1} + F_{n2}} \approx \frac{2}{c} h \sin(\phi_1)$$

(12)

As a result:

$$\phi_{target} = \pm \arcsin\left(\frac{0.8c}{2h}\right)$$

(13)

- Next, a desired reference trajectory $\phi_{Ref}$, joining the current state $\phi_{NL}$ to $\phi_{target}$ during the horizon of prediction is defined. Typically a first order discrete system is considered:

$$\phi_{Ref[n+i]} = \phi_{target} - \gamma \cdot (\phi_{target} - \phi_{NL[n]})$$

(14)

The subscripts $[n]$ and $[n+i]$ (with $0 \leq i \leq h$) denote respectively the current time instant $t$ and successive future time instants up to $t + H$ (since $[n+h]$ corresponds to time instant $t + H$) and $\gamma$ is a parameter tuning the settling time for the reference trajectory to reach the set point.

- Then, at each sample time, an optimal control sequence $(w_{[n]},...,w_{[n+h]})$ bringing $\phi_{NL}$ to $\phi_{target}$ is computed through the minimization of the quadratic criterion:

$$D_{[n]} = \sum_{i=1}^{h} \left( \phi_{NL[n+i]} - \phi_{Ref[n+i]} \right)^2$$

(15)

where $\phi_{NL[n+i]}$ denotes the predicted output process obtained from linear model (11) and the control sequence. The minimization can be achieved thanks to the decomposition of each element of the control sequence $(w_{[n+i]}, \ i \in [0,h])$ as a linear combination of base functions:

$$w_{[n+i]} = \sum_{k=1}^{n_B} \mu_{k[n]} w_{Bk[i]}, \quad 0 \leq i \leq h$$

(16)

where $\mu_{k[n]}$ are the coefficients supplied by the minimization of $D_{[n]}$, $n_B$ is the number of base functions and $w_{Bk}$ are the base functions, generally chosen as polynomials:

$$w_{Bk[i]} = t^{k-1}, \quad \forall k$$

(17)

If the optimal control sequence obtained from the minimization of $D_{[n]}$ was applied over the horizon of prediction, then $\phi_{NL}$ and $LLT$ would reach respectively $\phi_{target}$ and 0.8 at time $t + H$. Therefore, the first element of the control sequence, i.e. $w_{[n]}$, has to be considered as the maximum control input value, and then the maximum ATV velocity at sample time $[n]$ is $v_{max} = \sqrt{(w_{[n]})}$. Nevertheless, the linearization of equation (6) introduces some approximations that necessarily impair the accuracy of the predicted values of the roll angle and then of the $LLT$. In order to reduce the influence of these approximations, and then to refine $LLT$ prediction, one possibility consists in minimizing an extended criterion $D_{2[n]}$ incorporating the current and expected discrepancies between the roll angle values supplied by the nonlinear model (6) and the linear model (11):

$$D_{2[n]} = \sum_{i=1}^{h} \left( \phi_{NL[n+i]} + \hat{e}_{[n+i]} - \phi_{Ref[n+i]} \right)^2$$

(18)

where the future output error $\hat{e}_{[n+i]}$ is defined as:

$$\hat{e}_{[n+i]} = e_{[n]} = \phi_{NL[n]} - \phi_{NL[n]}, \quad 1 \leq i \leq h$$

(19)

Finally, the PFC algorithm comprises two parameters to be tuned: the gain $\gamma$ (specifying the shape of the reference trajectory) and the horizon of prediction $H$. 

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IV. RESULTS

In this section, advanced simulations are reported in order to validate the proposed control law dedicated to ATV dynamic lateral stability. First, the virtual quad bike built with the multibody dynamic software Adams is briefly described. Then, the performances of the PFC algorithm are investigated by using jointly Adams and Matlab/Simulink softwares.

A. Simulation testbed

A virtual quad bike (depicted on Fig. 5(a)) has been designed with dynamic multibody software Adams. With such softwares, body geometry, as well as joints and external forces are described without specifying explicit mechanical equations. The mechanical object thus obtained can then be supplemented with different ground models, and finally a purely numerical integration is performed in order to deliver the virtual vehicle’s motion. In previous work [5], it has been shown that the $LLT$ values obtained with this simulated vehicle were closely resembling to the ones recorded with our experimental vehicle shown on Fig.5(b). This virtual quad bike can therefore be considered as a very realistic testbed.

![Adams virtual quad bike.](image1)

![Experimental vehicle.](image2)

Fig. 5. Simulated and actual vehicles.

The quad bike parameters are listed in Table I. The first ones are directly inspired by the characteristics of our experimental vehicle shown on Fig.5(b). The two last ones, namely the roll stiffness $k_r$ and the distance $h$ between the roll center $O'$ and the center of gravity $G$ have been calibrated according to a first simulation run with a simulated high grip ground: $k_r$ and $h$ have been identified, via a Newton-Raphson non-linear algorithm, in order to minimize the difference between the computed $LLT$ and the $LLT$ supplied by Adams software (more details can be found in [3]).

<table>
<thead>
<tr>
<th>Virtual Quad Bike Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quad bike suspended mass</td>
<td>250 kg</td>
</tr>
<tr>
<td>$I_x, I_y, I_z$</td>
<td>45, 110, 130 kg.m$^2$</td>
</tr>
<tr>
<td>Front and rear half-wheelbases $a, b$</td>
<td>0.58, 0.7 m</td>
</tr>
<tr>
<td>Quad bike track $c$</td>
<td>0.95 m</td>
</tr>
<tr>
<td>Distance between $O'$ and $G$: $h$</td>
<td>1.24 m</td>
</tr>
<tr>
<td>Roll stiffness $k_r$</td>
<td>5900 N.m.rad$^{-1}$</td>
</tr>
</tbody>
</table>

B. PFC control results

1) Simulation parameters: The velocity and steering angle specified by the pilot during the simulation are depicted on Fig.6. The simulated grip conditions correspond to a wet grass soil. The velocity, the steering angle and the yaw rate have been recorded with the software Adams at a 100Hz frequency in order to emulate sensors.

![Pilot velocity.](image3)

![Steering angle.](image4)

Fig. 6. Velocity and steering angle specified by the pilot.

2) Velocity control: The rear axle linear velocity control strategy has been applied to the virtual quad bike. The horizon of prediction has been set to $H = 1s$ (according to the vehicle dynamic features), with 10 coincidence points (i.e. $h = 10$) and the reference trajectory has been computed with $\gamma = 0.2$. Fig.7 shows the time evolution of the velocity specified by the pilot $v_{pilot}$ (in blue dash-dotted line), the maximum velocity $v_{max}$ (computed with the PFC algorithm, in red solid line) and the rear axle velocity $v_{input}$ to be applied to the vehicle (in green dashed line). From $t = 0$ to $t = 10s$, the virtual quad bike is either moving according to a straight line or the steering angle value is low ($\delta < 3^\circ$). Therefore the maximum velocity cannot be computed and is then set to $14m/s$. After $t = 10s$, the velocity to be applied $v_{input}$ is equal to the minimum of $v_{pilot}$ and $v_{max}$, as described in Section III-A: first, $v_{input}$ is equal to $v_{pilot}$. Then, between $t = 27.6s$ and $t = 53.7s$, $v_{input} = v_{max}$, because $v_{pilot}$ is too high with respect to the steering angle value, even when the steering angle is decreased from $10^\circ$ to $7.5^\circ$. Finally, after $t = 53.7s$, $v_{pilot}$ has been reduced, so that $v_{pilot}$ can again be actually applied: $v_{input} = v_{pilot}$.

![Velocity control results.](image5)

Fig. 7. Velocity control results.

Fig.8 shows the time evolution of the $LLT$ measured on Adams when respectively $v_{input}$ (black solid line) and $v_{pilot}$ (red dashed line) are applied to the virtual quad bike. In the last case, after $t = 29s$, the vehicle rollovers (since $LLT = 1$). Contrarily, when $v_{input}$ is applied, the $LLT$ safely converges to the $LLT$ threshold value ($LLT = 0.8$) when $v_{pilot}$ exceeds
Vehicle rollover has then satisfactorily been avoided, while keeping vehicle velocity as high as possible in such a situation.

The simulation results reported in black solid line in Fig.8 have been obtained when relying on the extended criterion $D_{2[n]}$ in order to derive $v_{\text{max}}$. As explained in Section III-C, minimizing this criterion (rather than $D_{n}$) permits to reduce the influence of the approximations introduced when linearizing the roll angle model. In order to reveal the actual significance of $D_{2[n]}$ over $D_{n}$, Fig.9 shows the time evolution of the LLT measured on Adams when $v_{\text{max}}$ is derived by minimizing $D_{2[n]}$ (green solid line) or $D_{n}$ (red dashed line). It can be noticed that, when minimizing $D_{2[n]}$, the LLT converges accurately to the threshold value 0.8 between $t = 27.6s$ and $t = 53.7s$ (i.e. as long as $v_{\text{input}} = v_{\text{max}}$). In the contrary, when minimizing $D_{n}$, the LLT exceeds this threshold value (it converges to 0.9), so that the vehicle dangerously approaches a rollover situation. This shows the importance of incorporating output error compensation in PFC algorithm.

The validation of the proposed approach on actual quad bikes and mobile robots is under development. It will open the way to the development of on-board devices for ATV dynamic stability.

**REFERENCES**


